Mid-Term Exam: Mathematics (Differentiation) First Semester 2013-2014

Model Answer

[1]Limits:

(a) $\text{Lim}_{x\to 0} \frac{\sin 2x}{2^{3x}-1} = \frac{0}{0} = \text{Lim}_{x\to 0} \frac{\sin 2x}{2x} . \frac{3x}{2^{3x}-1} . \frac{2}{3} = 1. \frac{1}{\ln 2} . \frac{2}{3} = \frac{2}{3\ln 2}$

(b) $\text{Lim}_{x\to 2} \frac{\ln(x-1)}{x-2} = \frac{0}{0} = \text{Lim}_{(x-2)\to 0} \frac{\ln(1+(x-2))}{x-2} = 1$

(c) $\text{Lim}_{x\to\infty} \frac{2-3x+x^2}{x^3+2x-3} = \frac{\infty}{\infty} = \text{Lim}_{x\to\infty} \frac{\frac{2}{x^2}-\frac{3}{x}+1}{x+\frac{2}{x}-\frac{3}{x^2}} = \frac{0-0+1}{\infty+0-0} = \frac{1}{\infty} = 0$

--- -- 6 Marks

[2] Derivative

(a) $y = 2x^3 - 2\cos x^2,$ $\quad y` = 6x^2 + 4x \sin x^2$

(b) $\mathrm{y} = \sin^{-1}\mathrm{x}^2 + \tan^{-2}\mathrm{x}$, $\quad y` = \frac{2x}{\sqrt{1-x^4}} - 2(\tan x)^{-3}. sex^2 x$

(c) $\mathrm{y} = 4^{\mathrm{x}^3}.\tanh^{-1}\mathrm{x} + \log 8,$ $\quad y` = 4^{x^3}. \ln 4 \,.\, 3x^2 . \tanh^{-1} x + 4^{x^3} . \frac{1}{1-x^2} + 0$

(d) $y = \log \frac{\sqrt[4]{x+\cosh x}}{\sqrt[3]{\sinh x + \sec x}} = \frac{1}{4}\log(x+\cosh x) - \frac{1}{3}\log(\sinh x + \sec x)$

$$y` = \frac{1}{4}\frac{1}{\ln 10}\ \frac{1+\sinh x}{x+\cosh x} - \frac{1}{3}\frac{1}{\ln 10}\ \frac{\cosh x + \sec x . \tan x}{\sinh x + \sec x}$$

--- 8 Marks

[3]Prove that: $\tanh^{-1}\mathrm{x} = \frac{1}{2} \ln\frac{1+\mathrm{x}}{1-\mathrm{x}}$

If $y = \tanh^{-1}\mathrm{x} = \frac{1}{2}\ln\frac{1+\mathrm{x}}{1-\mathrm{x}}$

Then $\tanh \mathrm{y} = \frac{\sinh \mathrm{y}}{\cosh \mathrm{y}} = \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}} = \mathrm{x}$

Then $\frac{e^{2y}-1}{e^{2y}+1} = \mathrm{x}$ and $e^{2y} - 1 = \mathrm{x}(e^{2y}+1) = \mathrm{x}.e^{2y} + \mathrm{x}$

Then $e^{2y}(1-\mathrm{x}) = 1 + \mathrm{x}.$

Then $e^{2y} = \frac{1+\mathrm{x}}{1-\mathrm{x}}$

Then $2\mathrm{y} = \ln\frac{1+\mathrm{x}}{1-\mathrm{x}}$

Then $\mathrm{y} = \frac{1}{2}\ln\frac{1+\mathrm{x}}{1-\mathrm{x}}$

Then $\tanh^{-1}\mathrm{x} = \frac{1}{2}\ln\frac{1+\mathrm{x}}{1-\mathrm{x}}$

---2 Marks

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